Insights from the Theory of the Firm into Structural and Reduce Form Modeling

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Purpose

- Theoretically illustrate the distinction between structural and reduced form models in context of a firm's cost minimizing and profit maximizing decisions.
- Derive structural and reduced form estimating equations for a firm's cost minimizing and profit maximizing decision when the underlying technology is Cobb-Douglas.
- Discuss the challenges of parameter interpretation and hypothesis testing when attempting to answer common economic questions using structural and reduced form models.

Example of a Common Economic Question

You are interested in the relationship between fertilizer and seeding rates in US corn production. For example, you work for Corteva and want to know if recent and projected fertilizer price increases are going to increase the demand for corn seed, while decreasing the fertilizer demand.

In the form of a question please:

Does increasing fertilizer prices increase corn farmers' seeding rates?

Example Answer

How do you answer this question if you are a Prof in SWC up the hill?

- Partition a homogeneous field or fields into a bunch of small plots (you could also do this in a greenhouse for even more control over the growing environment).
- Randomly assign a treatment of fertilizer *F* and seeding rate *S* to each plot making sure that the same treatment *t* is replicated *r* times.
- Fertilize and seed according to your randomized experimental design.
- Harvest the corn in each plot at the end of the season and measure the bushels of corn produced.
- Run some regressions:

(1) $q_{tr} = \phi_1 + \sum_{t=2,..,T} \phi_t D_{rt} + \varepsilon_{tr}$

(2) $q_{tr} = \beta_0 + \beta_F z_{F_{tr}} + \beta_S z_{S_{tr}} + \beta_{FF} {z_{F_{tr}}}^2 + \beta_{SS} {z_{S_{tr}}}^2 + \beta_{FS} z_{F_{tr}} z_{S_{tr}} + \varepsilon_{tr}$

(3) $ln(q_{tr}) = \tau_0 + \tau_F ln(z_{F_{tr}}) + \tau_S ln(z_{S_{tr}}) + \varepsilon_{tr}$

where q is bushels of corn, Ds are treatment dummy variables, zs are the input quantities, and εs are random errors; ϕs, βs and τs are estimable parameters; and T is the total number of fertilizer and seeding rate treatments.

Comment:

- Equation (1) makes no assumptions about the functional relationship between bushels of corn, fertilizer, and seeding rates. ϕ_t measures the difference in the average bushels of corn between fertilizer and seeding treatments 1 and t.
- Equation (2) can be interpreted as a Taylor series approximation to a general differentiable functional form relating bushels of corn to fertilizer and seeding rates. If $\beta_{FS} > 0$, increasing fertilizer increases the marginal productivity of the seeding rate.
- Equation (3) can be interpreted as a Cobb-Douglas production function relating bushels of corn to fertilizer and seeding rates. If τ_F and τ_I in equation (3) are the same sign, increasing fertilizer increases the marginal productivity of the seeding rate.

Questions:

- How can we use these results to figure out if farmers will use a higher seeding rate with increased fertilizer prices?
- Which of these three models would you say is structural and which would you say is a reduced form?

Example Answer

How might you approach this question if you are a Prof in APEC?

- First, you might ask yourself what an appropriate behavioral model would be: cost minimization or profit maximization.
- Second, you need to determine the appropriate hypothesis for the question of interest given your behavioral model.
- Third, what data is available or needs to be collected to test your proposed hypothesis?
- Four, collect the data.
- Five, run regressions to test your hypothesis.

Potential Behavioral Models

Cost Minimization

(4) $\min_{\mathbf{z}\geq 0} \mathbf{r}\cdot\mathbf{z}$ subject to $(q,\mathbf{z}) \in PPS(q,\mathbf{z}|\varepsilon)$

where q is bushels of corn, $\mathbf{z}$ is an input vector, $\mathbf{r}$ is an input price vector, and ε measures idiosyncratic productivity.

The solution to this problem is a vector of input demands that depend on q, $\mathbf{r}$, and ε: $\mathbf{z}(q,\mathbf{r},\varepsilon)$. These input demands can then be used to formulate the cost function $C(q,\mathbf{r},\varepsilon) = \mathbf{r}\cdot\mathbf{z}(q,\mathbf{r},\varepsilon)$.

Profit Maximization

(5) $\max_{q\geq 0} pq - C(q,\mathbf{r},\varepsilon)$

where p is the price per bushel of corn.

The solution to this problem is the supply of corn that depends on p, $\mathbf{r}$, and ε: $q(p,\mathbf{r},\varepsilon)$. This supply can then be used to formulate the profit function $\pi(p,\mathbf{r},\varepsilon) = pq(p,\mathbf{r},\varepsilon) - C(q(p,\mathbf{r},\varepsilon),\mathbf{r},\varepsilon)$. Also recall that duality allows us to determine the vector of input demands that depend on p, $\mathbf{r}$, and ε: $\mathbf{z}(p,\mathbf{r},\varepsilon) = \mathbf{z}(q(p,\mathbf{r},\varepsilon),\mathbf{r},\varepsilon)$.

Definitions to Recall for Question at Hand

- Good i is a complement for good j if the demand for good i decreases when the price of good j increases or if the cross-price elasticity of demand is negative: $\xi_{ij(q,\mathbf{r},\varepsilon)} = \frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j}\frac{r_j}{z_i(q,\mathbf{r},\varepsilon)} < 0$ or $\xi_{ij(p,\mathbf{r},\varepsilon)} = \frac{\partial z_i(p,\mathbf{r},\varepsilon)}{\partial r_j}\frac{r_j}{z_i(p,\mathbf{r},\varepsilon)} < 0$.
- Good i is a substitute for good j if the demand for good i increases when the price of good j increases or when the cross-price elasticity of demand is positive: $\xi_{ij(q,\mathbf{r},\varepsilon)} = \frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j}\frac{r_j}{z_i(q,\mathbf{r},\varepsilon)} > 0$ or $\xi_{ij(p,\mathbf{r},\varepsilon)} = \frac{\partial z_i(p,\mathbf{r},\varepsilon)}{\partial r_j}\frac{r_j}{z_i(p,\mathbf{r},\varepsilon)} > 0$.

Note: Since that $z_i(q,\mathbf{r},\varepsilon) > 0$, $z_i(p,\mathbf{r},\varepsilon) > 0$ and $r_j > 0$, the inequalities really just depend of $\frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j}$ and $\frac{\partial z_i(p,\mathbf{r},\varepsilon)}{\partial r_j}$.

	Hypothesis C	**Hypothesis Π**
Null:	$\frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j} > 0$	$\frac{\partial z_i(p,\mathbf{r},\varepsilon)}{\partial r_j} > 0$
Alternative:	$\frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j} \leq 0$	$\frac{\partial z_i(q,\mathbf{r},\varepsilon)}{\partial r_j} \leq 0$

Adding More Structure: Cobb-Douglas

(6) $$PPS(q,\mathbf{z}|\varepsilon) = \left\{(q,\mathbf{z}) \in \mathbb{R}_+ \times \mathbb{R}_+^N \middle| q \leq \varepsilon \prod_{k=1}^{N} z_k{}^{\tau_k}\right\}$$

implies

(7) $$ln\big(z_n(q,\mathbf{r},|\varepsilon)\big) = \alpha_I^{z_n} + \alpha_{z_n} ln(r_n) + \sum_{j\neq n} \alpha_j ln\big(r_j\big) + \alpha_q ln(q) + \varepsilon_{lnC}$$

where

$\alpha_q = \frac{1}{\sum_{k=1}^{N} \tau_k}$,
$\alpha_I^{z_n} = ln(\tau_n) - \alpha_q \sum_{j=1}^{N} \tau_j \, ln\big(\tau_j\big)$,
$\alpha_q = \frac{1}{\sum_{k=1}^{N} \tau_k}$,
$\alpha_j = \alpha_q \tau_j$,
$\alpha_{z_j} = \alpha_j - 1$, and
$\varepsilon_{lnC} = -\alpha_q ln(\varepsilon)$.

Adding More Structure: Cobb-Douglas

(8) $$ln\left(q(p,\mathbf{r}|\varepsilon)\right) = \beta_I^q + \beta_p^q ln(p) + \sum_{j=1}^{N} \beta_j \, ln\left(r_j\right) + \varepsilon_{ln\pi}$$

where

$\beta_j = -\frac{\tau_j}{1-\sum_{k=1}^{N} \tau_k}$,
$\beta_I^q = -\sum_{j=1}^{N} \beta_j \, ln\left(\tau_j\right)$,
$\beta_p^q = -\sum_{j=1}^{N} \beta_j$, and
$\varepsilon_{ln\pi} = \frac{1}{1-\sum_{k=1}^{N} \tau_k} ln(\varepsilon)$.

Adding More Structure: Cobb-Douglas

(9) $$ln\left(z_n(p,\mathbf{r}|\varepsilon)\right) = \delta_I^{z_n} + \sum_{j=1,..,N} \delta_{z_j} ln\left(r_j\right) + \delta_p ln(p) + \varepsilon_{ln\pi}^{z_n}$$

where

$\delta_I^{z_n} = \alpha_I^{z_n} + \alpha_q \beta_I^q$,
$\delta_{z_n} = \alpha_{z_n} + \alpha_q \beta_n$,
$\delta_{z_j} = \alpha_j + \alpha_q \beta_j$,
$\delta_p = \alpha_q \beta_p^q$, and
$\varepsilon_{ln\pi}^{z_n} = \alpha_q \varepsilon_{ln\pi} + \varepsilon_{lnC}$.

Adding More Structure: Cobb-Douglas

Hypothesis C

Null: $\xi_{SF(q,\mathbf{r},\varepsilon)} = \alpha_F = \frac{\tau_F}{\sum_{k=1}^{N} \tau_k} > 0$

Alternative: $\xi_{SF(q,\mathbf{r},\varepsilon)} = \alpha_F = \frac{\tau_F}{\sum_{k=1}^{N} \tau_k} \leq 0$

Hypothesis Π

Null: $\xi_{SF(p,\mathbf{r},\varepsilon)} = \delta_{z_F} = \frac{\tau_F}{\sum_{k=1}^{N} \tau_k} - \frac{\tau_F}{\sum_{k=1}^{N} \tau_k \left(1 - \sum_{k=1}^{N} \tau_k\right)} = -\frac{\tau_F}{1 - \sum_{k=1}^{N} \tau_k} > 0$

Alternative: $\xi_{SF(p,\mathbf{r},\varepsilon)} = \delta_{z_F} = \frac{\tau_F}{\sum_{k=1}^{N} \tau_k} - \frac{\tau_F}{\sum_{k=1}^{N} \tau_k \left(1 - \sum_{k=1}^{N} \tau_k\right)} = -\frac{\tau_F}{1 - \sum_{k=1}^{N} \tau_k} \leq 0$

Adding More Structure: Cobb-Douglas

Data Requirements & Regression Methods
Equations (7) & (8)

- Seeding Rate
- All Input Prices
- Bushels of Corn
- Instrumental Variables Regression

Equations (9)

- Seeding Rate
- All Input Prices
- Price of Corn
- OLS

Which of these strategies would you consider structural and which would you consider reduced form?

Avoiding More Structure

(10) $$z_S = \lambda_0 + \lambda_F R_F + \mathbf{\Gamma X} + \boldsymbol{\varepsilon}$$

where R_F is a treatment variable capturing difference in the fertilizer price, **X** is a vector of covariates, **Γ** is a corresponding parameter vector, and **ε** is a vector of fixed effect and/or random errors.

Hypothesis R

Null: $$\frac{\partial z_S(q,\mathbf{r},\varepsilon)}{\partial r_F} = \lambda_F > 0$$

Alternative: $$\frac{\partial z_S(q,\mathbf{r},\varepsilon)}{\partial r_F} = \lambda_F \leq 0$$

Avoiding More Structure

Data Requirements & Regression Methods

Panel Data

- Seeding Rates Over Time
- Fertilizer Prices Over Time
- Other Information for **X**
- OLS with Farmer Fixed Effects

Regression Discontinuity

- Seeding Rates
- Difference in Fertilizer Price Attributable to Some Exogenous Factor (e.g., difference in tax policy between contiguous states)
- Treatment Dummy Variable based on Exogenous Factor (e.g., 1 for high fertilizer tax state, zero otherwise)
- Other Information for **X**
- OLS

Avoiding More Structure

Data Requirements & Regression Methods

Randomized Control Trials

- Seeding Rates
- Fertilizer Prices
- Randomly Assigned Fertilizer Price Discount
- Other Information for **X**
- OLS

Other Quasi Experimental Designs

- Seeding Rates
- Fertilizer Prices
- Exogenous Shock to Fertilizer Prices
- Other Information for **X**
- OLS

Questions or Comments