

# Creating Theory in Practice: An Application from Environmental/Welfare Economics

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- The abatement cost function is a workhorse tool in environmental economics
- Theoretical models in a huge number of papers reduce a firm's operations to a simple abatement cost function
- The cost of abatement depends only on emissions  $e_i$  or on abatement  $a_i = E_i^0 - e_i$ :  
$$C_i(e_i) \quad \text{or} \quad C_i(a_i)$$
- Industries are often characterized similarly:  $C(e)$  or  $C(a)$
- In this framing, reducing emissions is costly:  $C'(a) > 0$

- “All price-taking firms attempt to minimize the sum of abatement costs and permit costs.”
- “Let  $C_i(Q_i)$  be the abatement cost associated with emitting  $Q_i$  units.”

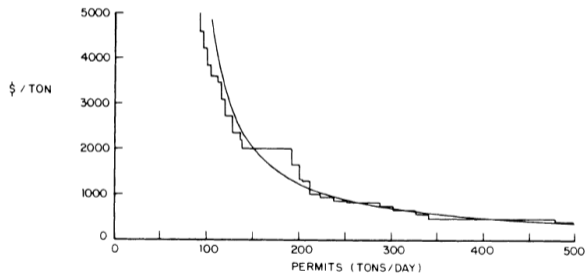


FIGURE II  
Derived Demand for Permits by All Other Firms

Suppose instead that the environmental agency pursues an IB strategy. By this we mean that it seeks that vector of emissions ( $E_1$ ) that can attain the standard at the minimum aggregate abatement cost:

$$\text{Min } \Sigma C(e_i)$$

$$\text{s.t. } ED \leq Q^*$$

$$E \geq 0,$$

where  $Q^*$  is the upper bound on allowable pollutant concentrations. There are various

Let abatement by firm  $j$  in period  $t$  be  $q_t^j$ . The abatement cost function for firm  $j$  is given by  $C^j(q_t^j, \theta)$ . Here,  $\theta$  is a realization of a random variable  $\Theta$ , which is known to the firms but not known by the regulator. We assume that the marginal abatement cost function for each firm is positive and increasing in abatement ( $C_q^j(q_t^j, \theta) > 0$  and  $C_{qq}^j(q_t^j, \theta) > 0$ ) and that  $C^j(0, \theta) = 0$ .

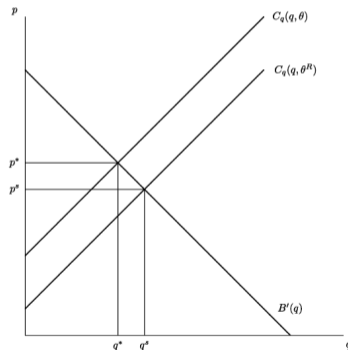


Fig. 1. Steady-state abatement, emissions taxes.

# The genesis of a paper idea

- In a conversation about the EU's proposal to require all airlines serving member countries to participate in the Union's carbon trading scheme,
- My Swedish interlocutor observed that U.S. airlines should not have fought the requirement
- They could increase their profits by reducing emissions, he claimed
- What? Everyone knows abatement is costly. I was skeptical

# Aviation and the EU's Directive 2008/101/EC



- On November 27, 2012, President Obama signed

Public Law 112–200  
112th Congress

## An Act

To prohibit operators of civil aircraft of the United States from participating in the European Union’s emissions trading scheme, and for other purposes.

*Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,*

### **SECTION 1. SHORT TITLE.**

This Act may be cited as the “European Union Emissions Trading Scheme Prohibition Act of 2011”.



# The genesis of a paper idea

- But *could* a new or tighter restriction on emissions make firms in the polluting industry better off?
- This would violate the polluter-pays principle
- Also, I observed, wouldn't profit-maximizing competitive firms already have done it?

# Windfall profits are actually possible

- An evening conversation with a smart friend made me reconsider
- Under certain conditions, I realized, windfall profits are possible
- I took the question to a research group and they liked it
- This lecture is the story of how we produced a theory paper exploring the idea
- Including some difficulties encountered along the way and how we met them



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## Price Effects, Inefficient Environmental Policy, and Windfall Profits

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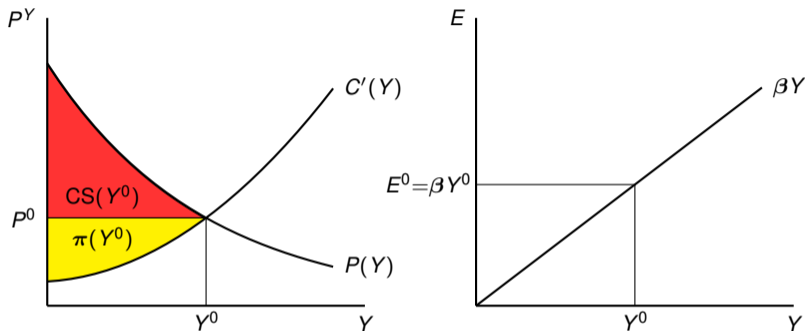
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# I understand analytical problems graphically

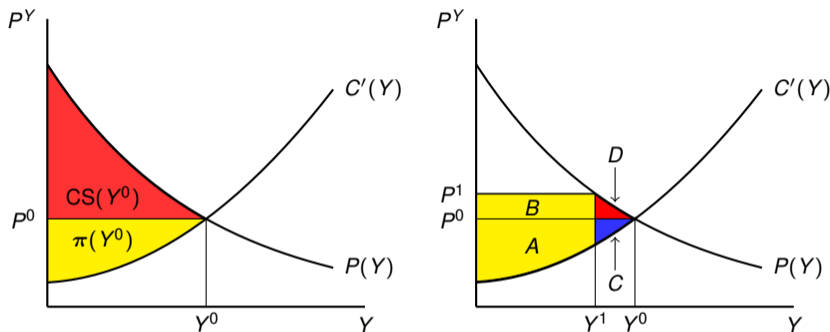
- That evening conversation involved sketching some diagrams
- The most straightforward case includes some restrictive conditions
  - The only way to reduce emissions is by reducing output
  - Marginal cost of producing output is strictly increasing
  - The required reduction in emissions is not too large

# An industry is in equilibrium with emissions $E^0$ , output $Y^0$



- $C'(Y)$  is industry supply
- Profits are strictly positive
- Emissions are proportional to output  $Y$

# Emissions limit imposed at $E^1 < E^0$

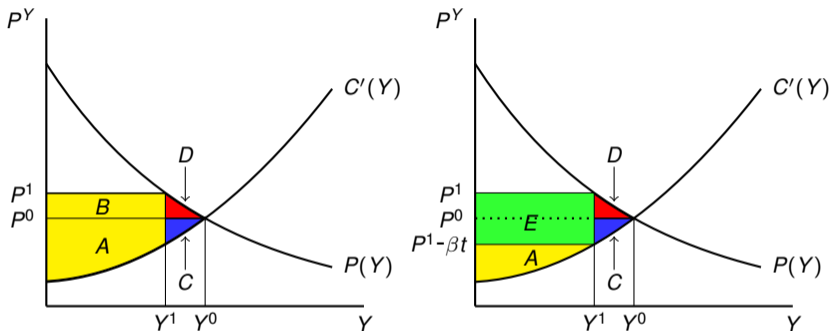


- To clear the market at  $Y^1$ , the output price must increase; the *price effect*
- Area  $B$  is an increase in profit,  $C$  is a decrease
- Windfall profits are definitely possible with a quantity policy

## But other questions occurred to us

- What if the policy is a tax on emissions?
- What if emissions are proportional to a polluting input, like coal?
- What if the EPA fails to account for the price effect at work? Might a welfare-maximizing EPA then choose the wrong emissions target?

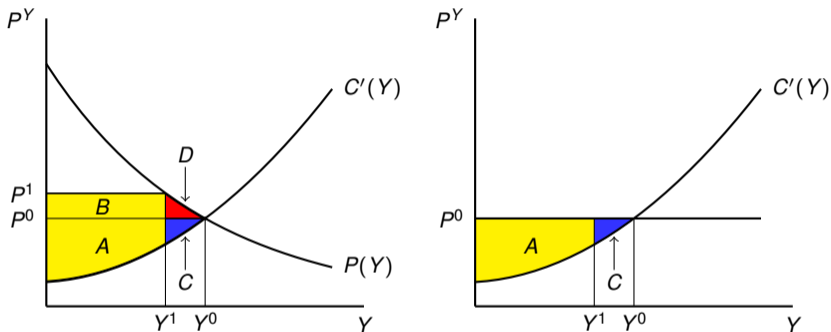
# Emissions tax. No windfall profits



- Under further conditions, an emissions tax cannot generate windfall profits
- It looks obvious, but nailing down this result was stubbornly difficult

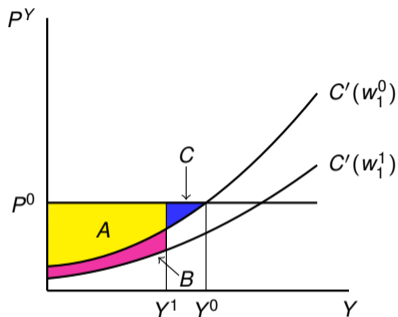
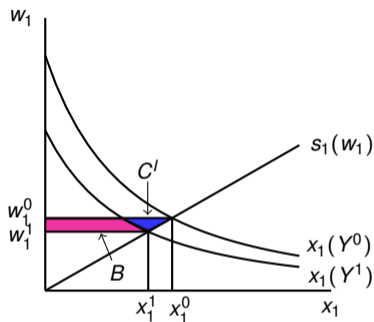


# No output price effect. No windfall profits



- If the output price is fixed, again no windfall profits
- This is the condition that is implicitly assumed in many papers

# Quantity restriction, input effect. Windfall profits $> 0$



- If emissions are proportional to a polluting input, windfall profits possible
- This result appears to be new in the literature
- It required the most subtle proof

# Why do so many ignore price effects?

- Perhaps due to Montgomery, *JET*, 1972
- Competitive firms, cost function  $G_i(y_{i1}, \dots, y_{iR}, e_i)$

- Max profits with unconstrained emissions:

$$\pi_i = \sum_r p_r \bar{y}_{ir} - G_i(\bar{y}_{i1}, \dots, \bar{y}_{iR}, \bar{e}_i)$$

- Max profits with emissions constrained to  $e_i$

$$\tilde{\pi}_i = \sum_r p_r \tilde{y}_{ir} - G_i(\tilde{y}_{i1}, \dots, \tilde{y}_{iR}, e_i)$$

- Cost to firm  $i$  of adopting emission level  $e_i$  is

$$F_i(e_i) = \pi_i - \tilde{\pi}_i$$

- This is Montgomery's abatement cost function and it is ubiquitous

# Montgomery was careful with assumptions

- “In a certain region there is a set of  $n$  industrial sources of pollution, each of which is fixed in location and owned by an independent, profit-maximizing firm.”
- “The prices of the inputs and outputs of these firms are fixed, **because the region is small relative to the entire economy.**”
- “Therefore any change in the level of output of a firm or industry in the region will have only a negligible impact on the output of the economy as a whole, and prices will be unaffected by output changes in the region.”
- Tension between large region (competitive permit market) and small region (no output price effect)

We specifically assume that

- (i) in the permit trading cases, the market price of permits falls when the marginal costs of firms in the industry decline;
- (ii) all firms in the industry adopt the new technology;
- (iii) the size and cost structure of the industry and the level of industry emissions are the same at the outset regardless of the policy instrument the industry faces;<sup>6</sup>
- (iv) firms do not exit or enter the industry during the period in question;
- (v) the market price of the industry's output does not change after the technology adoption.<sup>7</sup>

<sup>7</sup> This situation would be met if the demand for the industry's output is infinitely elastic.

### 3. The model and some basic results

Consider a competitive industry consisting of  $n$  small firms where industry size is assumed to be exogenous. All the firms emit a homogeneous pollutant which can be abated. The firms' abatement cost functions  $C_i$  satisfy the usual properties:  $C_i(e_i) > 0$  for  $e_i < e_i^{\max}$ , and  $C_i(e_i) = 0$  for  $e_i \geq e_i^{\max}$ , where  $e_i^{\max}$  denotes the maximal emission level under *laissez-faire*. Moreover, marginal abatement costs are positive in the relevant range, i.e.  $-C'_i(e_i) > 0$  for  $e_i < e_i^{\max}$ , and decreasing with more emissions, i.e.  $C''_i(e_i) > 0$  for  $e_i < e_i^{\max}$ . Since the product market is assumed to be competitive, decisions about output need not be modelled explicitly. Those decisions are implicitly accounted for in the abatement cost functions.

# Our model. Notation and key assumptions

- Competitive industry with  $n$  identical polluting firms,  $i = 1, \dots, n$
- Firm  $i$  uses  $m$  inputs  $\mathbf{x}^i = (x_1^i, \dots, x_m^i)$ , purchased in competitive markets at prices  $\mathbf{w}$
- To produce a single output according to the production function  $y^i = f^i(\mathbf{x}^i)$
- $f^i$  has continuous third derivatives and satisfies  $\partial^2 f^i / \partial x_j \partial x_k \geq 0$  for  $j \neq k$
- A single pollutant is emitted at a fixed rate  $e^i = \beta y^i$

<sup>7</sup> Emissions are a fixed proportion of output in Buchanan and Tullock (1975), Aidt and Dutta (2004), Mansur (2009), Perino (2010), Coria (2009) and Bréchet and Jouvet (2008).

# Our model. Notation and key assumptions

- Input  $x_1$  is a polluting input, coal for example
- The industry is small relative to its input markets, so all input prices, including  $w_1$ , are fixed
- Later, we assume the polluting industry is a large buyer of  $x_1$ , and  $w_1$  changes
- Demand for  $Y$  is  $P(Y)$ , twice differentiable and downward sloping



# Our model. Notation and key assumptions

- The minimum cost to firm  $i$  of producing  $y^i$  is

$$C^i(y^i, \mathbf{w}) = \min_{\mathbf{x}^i} \{ \mathbf{w}\mathbf{x}^i \mid y^i = f^i(\mathbf{x}^i) \}$$

- Cost is increasing and, because  $f^i$  is strictly concave,  $C^i$  is strictly convex in  $y^i$
- Industry costs  $C(Y)$  are the sum of the  $C^i(y^i)$  across firms
- Industry marginal cost, or inverse supply, is  $C'(Y)$
- It is the short run, so there is no entry or exit

# The initial situation

- In the initial situation there is no environmental restriction
- Firms take prices as given and choose  $y^i$  to maximize

$$\pi^i = Py^i - C^i(y^i, \mathbf{w})$$

- Initial equilibrium output is  $Y^0 = ny^{i0}$ , where  $P(Y^0) = C'(Y^0)$
- The initial price is  $P^0 = P(Y^0)$
- Initial uncontrolled emissions are  $E^0 = \beta Y^0$
- Initial profits are  $\pi^{i0}$  for each firm and  $\pi^0 = n\pi^{i0}$  for the industry
- Our main interest is in how  $\pi^0$  changes in response to a policy

## Two policy options

- First: a quantity restriction  $r \in [0, 1]$  on emissions
- Firm emissions must fall to  $e^{i1} = re^{i0}$ , aggregate emissions to  $E^1 = rE^0$
- Firm output must fall to  $y^{i1} = ry^{i0}$  and industry output to  $Y^1 = rY^0$
- To clear the market, output price must rise to  $P(Y^1)$
- Industry profits at  $r$  are denoted  $\pi(r)$
- Change in industry profits from the quantity regulation is  $\Delta\pi(r) = \pi(r) - \pi(1)$

# Two policy options

- Second: a tax  $t$  on each unit of emissions
- This is equivalent to a tax of  $\beta t$  on each unit of output
- For any  $r$ , the equivalent emissions tax will solve

$$t(r) = \frac{P(rY^0) - C'(rY^0)}{\beta}$$

- Industry profits at  $r$  are denoted  $\pi(t)$
- Change in industry profits from the tax is  $\Delta\pi(t) = \pi(t) - \pi(0)$

# Damages, social welfare, and optimal policy

- Monetary damage function is  $D(E)$  with  $D(0) = 0$
- We assume  $D(E)$  is differentiable, increasing, and strictly convex in  $E$
- This means marginal damages,  $D'(E)$ , are increasing in emissions
- We also explore how a regulator chooses policy to minimize social cost plus damages

# Where are we so far?

- The basic analytical framework has been established
- A bunch of assumptions have been imposed
- Some of these are more objectionable than others. We defend them, usually in footnotes
- Some are obvious, others less so
- For example, why do we need the third derivative of  $f^i$  to be continuous?
- And why do we need  $\partial^2 f^i / \partial x_j \partial x_k \geq 0$ ?

# An aside regarding assumptions

- Never impose an assumption that you don't use explicitly!
- Those two are a little unusual: continuous third derivative and nonnegative cross partial of  $f^i$
- They will stick out to reviewers. They better be important
- It's time to state and prove some propositions

# The first proposition

**Proposition 1** *Consider a competitive polluting industry satisfying the assumptions laid out above. Suppose industry demand  $P(Y)$  is downward sloping and industry marginal cost  $C'(Y)$  is upward sloping in  $Y$ , and that all input prices are fixed.*

- (i) There is  $\tilde{r} < 1$  such that  $\Delta\pi(\tilde{r}) = 0$ . If industry profit is strictly concave in  $r$ ,  $\tilde{r}$  is unique, and  $\Delta\pi(r) > 0$  for  $r \in (\tilde{r}, 1)$ .<sup>10</sup>*
- (ii) For any  $t > 0$ ,  $\Delta\pi(t) < 0$ .*
- (iii) In the absence of output price effects, with the output price fixed at  $P^0$ ,  $\Delta\pi < 0$  for any  $r < 1$  and for any  $t > 0$ .*



## Appendix

*Proof of Proposition 1 (i)* We first show that  $\partial\pi(r)/\partial r < 0$  when evaluated at  $r = 1$ . In the initial situation, equilibrium requires that  $P(Y^0) = C'(Y^0)$ , so that  $E^0 = \beta Y^0$ . With regulation at  $r < 1$ , emissions are  $E^1 = rE^0 = r\beta Y^0$ , output drops to  $Y^1 = rY^0$ , and aggregate profits become  $\pi(r) = P(rY^0)rY^0 - C(rY^0)$ . The change in profits in response to  $r$  is

$$\frac{\partial\pi(r)}{\partial r} = Y^0 [P'(rY^0)rY^0 + P(rY^0) - C'(rY^0)]. \quad (11)$$

Evaluating (11) at  $r = 1$ , and using  $P(Y^0) = C'(Y^0)$  and  $P'(Y) < 0$ , we can see that  $\partial\pi(1)/\partial r < 0$ . It follows that profits must increase as  $r$  moves left from 1, and thus that there exists  $r < 1$  at which  $\Delta\pi > 0$ . Because  $C(0) = 0$ , we must have  $\pi(0) = 0$ , which in turn means that  $\Delta\pi(0) < 0$ . By the intermediate-value theorem there is  $\tilde{r} \in (0, 1)$  at which  $\Delta\pi(\tilde{r}) = 0$ .

To see that  $\tilde{r}$  is unique, note that the second derivative is

$$\frac{\partial^2\pi(r)}{\partial r^2} = (Y^0)^2 [2P'(rY^0) + rP''(rY^0)Y^0 - C''(rY^0)] < 0.$$

This inequality, which appears in footnote 10, ensures that profits are strictly concave in  $r$ . Thus,  $\tilde{r}$  is unique. The strict concavity of  $\pi(r)$  ensures in turn that  $\Delta\pi(r) > 0$  for  $r \in (\tilde{r}, 1)$ .

# Welfare loss if policy ignores price effects

- We've now established when windfall profits occur and when they don't
- Next we turn to the question of social welfare
- Social cost of abatement, at policy  $r$ :

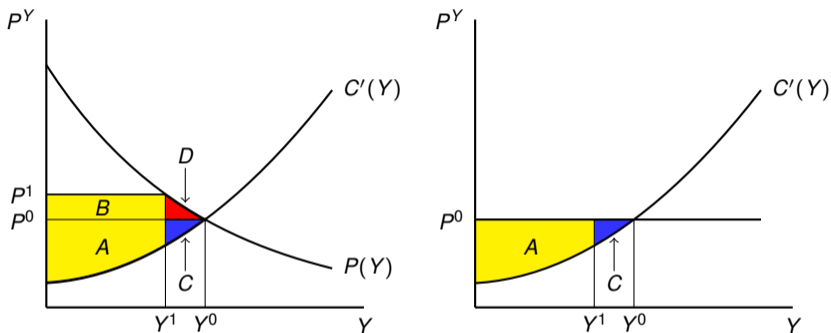
$$L^{\text{Out}}(r) = \int_r^1 [P(zY^0) - C'(zY^0)] dz,$$

- Where "Out" refers to the output price effect
- The truly optimal policy  $r^*$  minimizes  $L^{\text{Out}}(r) + D(rE^0)$
- A regulator who ignores price effects mistakenly thinks social cost is

$$\hat{L}(r) = \int_r^1 [P^0 - C'(zY^0)] dz,$$

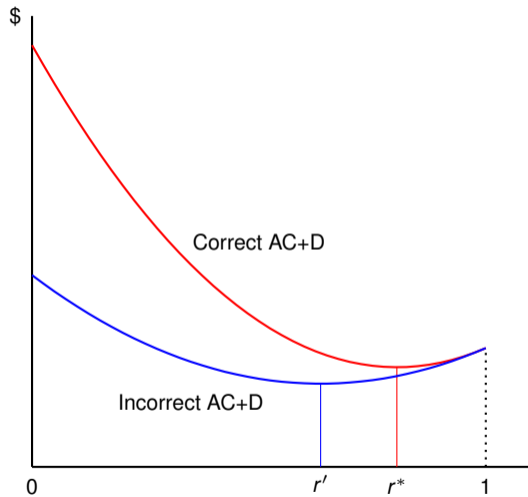
- The hapless regulator chooses  $r$  to minimize  $\hat{L}(r) + D(rE^0)$
- These objective functions lead to different policy choices

# Correct and incorrect social cost measures



- Correct social cost of abatement  $L^{\text{Out}}(r)$  is  $C + D$
- Incorrect social cost of abatement  $\hat{L}(r)$  is just  $C$
- A regulator who ignores  $D$  chooses too much abatement

# Welfare loss if policy ignores price effects



# Proposition 2 and proof

**Proposition 2** *If a regulator seeking to maximize social welfare ignores output price effects, the chosen level of abatement will be too high for either policy:  $\hat{r} < r^*$  and  $\hat{t} > t^*$ .*

*Proof of Proposition 2* The regulator who accounts properly for the output price effect will choose  $r$  to minimize the sum of damages and  $L^{\text{Out}}(r)$  from (2). The solution is  $r^*$  satisfying the first-order necessary condition

$$\frac{\partial L^{\text{Out}}(r^*)}{\partial r} + E^0 D'(r^* E^0) = -P(r^* Y^0) + C'(r^* Y^0) + E^0 D'(r^* E^0) = 0. \quad (13)$$

The regulator who ignores the output price effect minimizes the sum of damages and  $\widehat{L}(r)$  from (3). This solution is  $\hat{r}$  satisfying

$$\frac{\partial \widehat{L}(\hat{r})}{\partial r} + E^0 D'(\hat{r} E^0) = -P^0 + C'(\hat{r} Y^0) + E^0 D'(\hat{r} E^0) = 0. \quad (14)$$

Let  $G_1(r)$  and  $G_2(r)$  denote the right-hand sides of (13) and (14):

$$G_1(r) = -P(r Y^0) + C'(r Y^0) + E^0 D'(r E^0) \quad \text{and} \quad (15)$$

$$G_2(r) = -P^0 + C'(r Y^0) + E^0 D'(r E^0). \quad (16)$$

Because  $P(r Y^0) \geq P^0$  for any  $r \in [0, 1)$ , it follows that  $G_1(r) < G_2(r)$  for any  $r \in [0, 1)$ . Thus,  $G_2(r^*) > G_1(r^*) = 0 = G_2(\hat{r})$ . By the strict convexity of  $C(\cdot)$  and  $D(\cdot)$ , we also know that  $G_2(r)$  is strictly increasing in  $r$ . Thus, the previous inequality also implies that  $\hat{r} < r^*$ , as was to be shown.

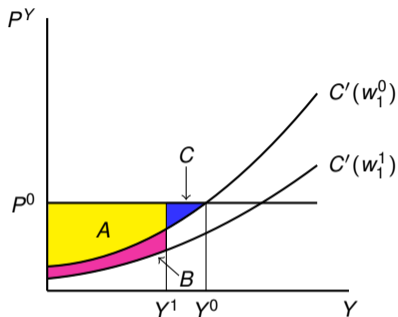
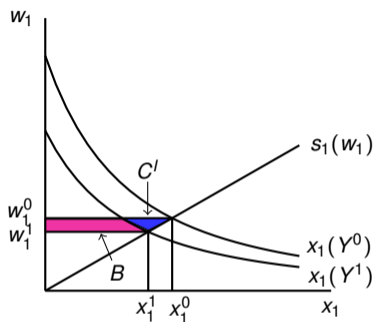
## Input price effect is a little different

- Our industry is the only buyer of  $x_1$ , with price  $w_1$
- As  $Y$  falls, demand for  $x_1$  shifts left and the equilibrium  $w_1$  also falls
- This can increase industry profits too, even with  $P$  fixed
- Individual firm demand for inputs is
$$\mathbf{x}^i(y^i, \mathbf{w}) = \arg \min_{\mathbf{x}^i} \{ \mathbf{w}\mathbf{x}^i \mid y^i = f^i(\mathbf{x}^i) \}$$
- Firm's demand for  $x_1$  is  $x_1^i(y^i, w_1)$ , cost function is  $C^i(y^i, w_1)$

# Input price effect is a little different

- The key: input price response shifts marginal cost down
- Windfall profits definitely occur with a quantity policy
- With output price effect, rents flow from consumers to polluters
- Now rents flow from suppliers of  $x_1$  to polluters

# Input price effect with a quantity policy: windfall $> 0$



- We can evaluate windfall profits in the input space (left)
- Or in the output space (right)
- Which is right? Turns out the two pink areas are the same



# Proposition 3

**Proposition 3** *Suppose the output price is fixed and consider an industry in which cost functions and input supply satisfy the conditions imposed above. The change in industry profit resulting from a change in  $w_1$  is equal whether it is measured in the input domain or the output domain:*

$$\int_{w_1^1}^{w_1^0} x_1(Y^1, z) dz = \int_0^{Y^1} [C_Y(z, w_1^0) - C_Y(z, w_1^1)] dz. \quad (6)$$

- Proof relies upon Shephard's lemma:  $C(Y, w_1) = \int x_1(Y, z) dz$

## Proposition 4

**Proposition 4** *Suppose that the output price  $P^0$  is fixed, industry marginal cost  $C_Y(Y, w_1)$  is strictly increasing in  $Y$ , the supply curve for  $x_1$  is increasing in  $w_1$ , and  $x_1$  is a normal input.*

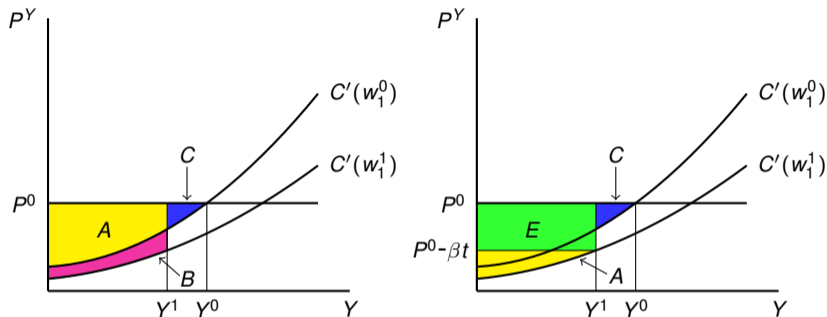
(i) *There is  $\tilde{r} < 1$  such that  $\Delta\pi(\tilde{r}) = 0$ . If in addition*

$$\frac{\partial x_1(Y, w_1)}{\partial w_1} < \frac{C_{YY} + 2C_{Yw_1} + x_1(Y, w_1)(\partial^2 w_1 / \partial Y^2)}{(\partial w_1 / \partial Y)^2}, \quad (8)$$

*then  $\tilde{r}$  is unique and  $\Delta\pi(r) > 0$  for  $r \in (\tilde{r}, 1)$ .*

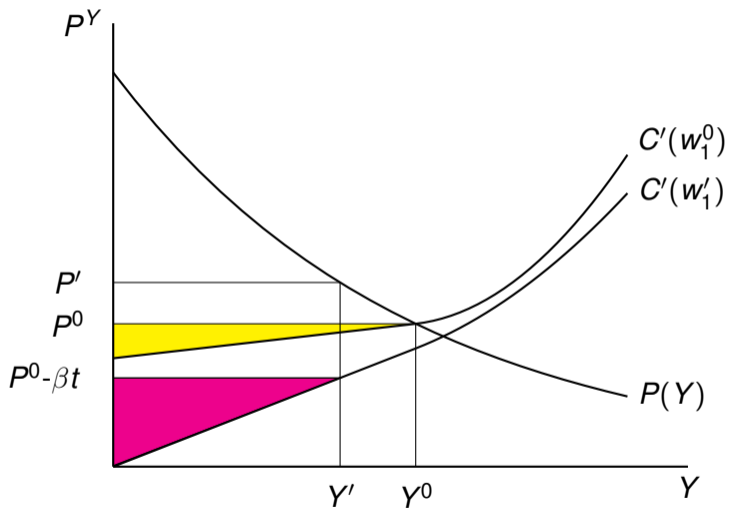
- The proof for the quantity policy is fairly straightforward
- This is where we need  $x_1$  to be a normal input, which it is if  $\partial^2 f^i / \partial x_j \partial x_k \geq 0$

# Input price effect with a tax policy: more subtle



- Our instincts said the right figure must be correct
- The tax causes  $x_1$  to fall, so  $w_1$  falls, so  $C'$  shifts down. Profit must fall too
- The following figure gave us fits, and cost us months

# A weird case where a tax causes windfall profit



# How to rule out this possibility?

- The problem is that marginal cost gets steeper as  $w_1$  falls
- We needed a condition on  $C(\cdot)$  that rules this out
- That is, we need to be sure that

$$\frac{\partial^3 C}{\partial w_1 \partial^2 Y} > 0 \quad (1)$$

- The trick was to see that, from Young's theorem and Shephard's lemma, we have

$$\frac{\partial^3 C}{\partial^2 Y \partial w_1} = \frac{\partial^2 x_1(Y, w_1)}{\partial Y^2} > 0$$

- So we just assumed (1) is true
- And this is why we also assumed  $f$  has a continuous third derivative

# Proposition 4

**Proposition 4** *Suppose that the output price  $P^0$  is fixed, industry marginal cost  $C_Y(Y, w_1)$  is strictly increasing in  $Y$ , the supply curve for  $x_1$  is increasing in  $w_1$ , and  $x_1$  is a normal input.*

(i) *There is  $\tilde{r} < 1$  such that  $\Delta\pi(\tilde{r}) = 0$ . If in addition*

$$\frac{\partial x_1(Y, w_1)}{\partial w_1} < \frac{C_{YY} + 2C_{Yw_1} + x_1(Y, w_1)(\partial^2 w_1 / \partial Y^2)}{(\partial w_1 / \partial Y)^2}, \quad (8)$$

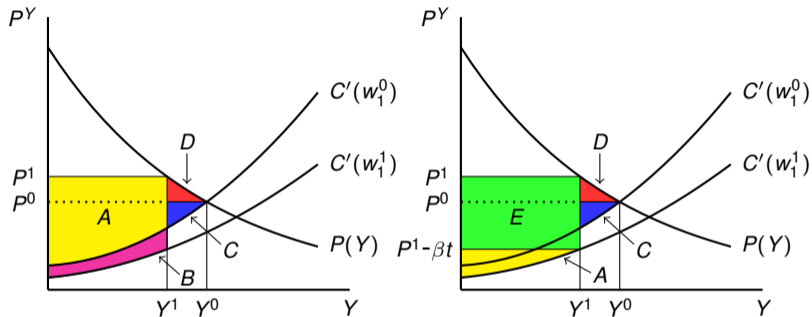
*then  $\tilde{r}$  is unique and  $\Delta\pi(r) > 0$  for  $r \in (\tilde{r}, 1)$ .*

(ii) *Assume that*

$$\frac{\partial C_{YY}}{\partial w_1} > 0. \quad (9)$$

*Then  $\Delta\pi(t) < 0$  for any  $t > 0$ .*

# Quantity restriction or emissions tax, both effects



- This part of the paper occupies one page, and no math
- We claimed that nothing new happens with both effects. Reviewers bought it