# Shift-Share Instrumental Variables

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APEC Skills Workshop

# Plan for Today

- What is a shift-share IV?
- What are the identification assumptions?
- How do I implement it?

**This hour will be more conceptual than mathematical**

# What is a shift-share IV?

# The first time I saw SSIV

- "Dams" by Esther Duflo and Rohini Pande (2005)
  - What is the impact of dams on poverty?
  - Idea: instrument dam incidence with river gradient
- Problem: $D_{ist} = \alpha + \beta RG_i + \gamma_i + \mu_{st} + \epsilon_{ist}$
- Solution: $D_{ist} = \alpha + \beta(RG_i \times \overline{D_{st}}) + \gamma_i + \mu_{st} + \epsilon_{ist}$

$$\text{where } \underbrace{\overline{D_{st}} = D_t \times share_{s(t=1970)}}_{SSIV}$$

# What is a shift-share IV?

- Weighted sum of common shocks with weights equal to heterogeneous exposure shares

$$z_l = \sum_n g_n s_{ln}$$

- Shocks vary at a different "level", $n = 1, .., N$ than the shares $l = 1, ..., L$, where we also observe outcome, $y_l$, and treatment, $x_l$.
- Goal: use $z_l$ to estimate causal treatment effects
- e.g. estimate parameter $\beta$ from model $y_l = \beta x_l + \epsilon_l$

**Under what assumption is $\beta$ identified?**

# Example I: Blanchard and Katz (1992)

Instrument $z_l = \sum_n \underbrace{g_n}_{shift} \underbrace{s_{ln}}_{share}$ for model $y_l = \beta x_l + \gamma' w_l + \epsilon_l$

- $\beta =$ inverse local labor supply elasticity
- $x_l =$ employment growth in region $l$
- $y_l =$ wage growth in region $l$

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- $\beta$ = inverse local labor supply elasticity
- $x_l$ = employment growth in region $l$
- $y_l$ = wage growth in region $l$
- need a labor demand shifter as an IV
- $g_n$ = national growth of industry $n$
- $s_{ln}$ lagged employment share (of industry $n$ in region $l$)
- $z_l$ = predicted employment growth due to national industry trends

# Example II: Imbert et al. (2022)

Instrument $z_l = \sum_n \underbrace{g_n}_{shift} \underbrace{s_{ln}}_{share}$ for model $y_l = \beta x_l + \gamma' w_l + \epsilon_l$

- $\beta$ = impact of migrant inflows on firm productivity
- $x_l$ = migration into region $l$
- $y_l$ = firm outcome region $l$

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- $\beta$ = impact of migrant inflows on firm productivity
- $x_l$ = migration into region $l$
- $y_l$ = firm outcome region $l$
- need an IV for migrant flows
- $g_n$ = shock to ag incomes in origin $o \in \Omega/n$
- $s_{ln}$ settlement patterns of past migration from $l$ to $n$
- $z_l$ = predicted migrant inflows to region $l$ due to ag shocks in origin

# Building Blocks of SSIV

- The shift-share does not need to be cleverly "made up"
- First, decompose treatment variation into space and time
- Let $X_{lt}$ be crop output in county $l$ for $t = 0, 1$; $x_l = \frac{X_{l1} - X_{l0}}{X_{l0}}$ is crop growth
- We can further decompose local production over crops $n$ as follows:

$$x_l = \sum_n \underbrace{\frac{X_{ln0}}{X_{l0}}}_{\text{local share}} \cdot \underbrace{x_{ln}}_{\text{local shift}} \qquad \text{where} \quad x_{ln} = \frac{X_{ln1} - X_{ln0}}{X_{ln0}}$$

- local shifts reflect crop supply and demand – need to isolate supply variation

**Construct SSIV by choosing common shift $g_k$ to replace $x_{lt}$, e.g. $DegreeDays_{ltn}$**

# SSIV Validity Conditions

- Once you have found a $z_l$, think through the validity condition
- Standard IV assumption $E[z_l \epsilon_l] = 0$ does not hold!

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- Once you have found a $z_l$, think through the validity condition
- Standard IV assumption $E[z_l \epsilon_l] = 0$ does not hold!
- Why? SSIV is a weighted sum of common shocks $\rightarrow$ observations not i.i.d
- SSIV validity condition is $E[\frac{1}{L}\sum_l z_l \epsilon_l]$
- Dividing by L ensures asymptotic consistency

$$\frac{1}{L}\sum_l z_l \epsilon_l \xrightarrow{LLN} E[z_l \epsilon_l]$$

**Takeaway: ensure exogeneity at aggregate level, not individual level**

# Summary of Validity Conditions

| Feature | Standard IV | SSIV |
|---|---|---|
| **Exogeneity Condition** | $E[z_\ell \varepsilon_\ell] = 0$ | $E\left[\frac{1}{L}\sum_\ell z_\ell \varepsilon_\ell\right] = 0$ |
| **Why?** | Each unit gets an independent instrument | SSIV aggregates shocks across many units |
| **Key Mechanism** | Randomization or exclusion restriction at the unit level | Averaging over many shocks ensures exogeneity |
| **What happens as $L \to \infty$?** | The instrument remains valid for each unit | Exogeneity holds in expectation over all units |

What properties of shifts and shares make this condition hold?

# Identification I: Shares are Exogenous

- Developed by Goldsmith-Pinkham et al. (2020), AER
- Imagine $s_{ln}$ is randomly assigned to units and exclusion restriction holds
  - Implies that, without treatment, units with different exposures would have trended similarly
- Equivalent to pooling diff-in-diff for each industry $n$
- i.e., a unit hit more by the shock, as captured by its randomly-assigned $s_{ln}$, would have trended similarly if it were non-exposed
- Must assume no unobserved shocks that affect outcome via same shares

# Example: Mariel Boatlift (Card, 1990)

- Goal: Estimate $\beta$ = elasticity b/w migrant vs. native workers in labour demand

$$y_l = \beta x_l + \gamma' \omega_l + \epsilon_l$$

- $y_l$ = relative wages, $x_l$ = rel. employment in location $l$
- Shift: sudden inflow of Cuban immigrants; Share: lagged Cuban workers in $l$
- Must assume regions more/less exposed to inflow (based on shares) have parallel trends in demand for migrant vs. native labour
- Equivalent to SSIV with Cuban inflows = 1, other countries = 0
- SSIV compares before/after shock, across cuban shares, and pools over all origins

# More formally...

**Step 1: Start with the SSIV Instrument**

$$E\left[\sum_\ell z_\ell \varepsilon_\ell\right] = E\left[\sum_\ell \sum_n s_{\ell n} g_n \varepsilon_\ell\right]$$

**Step 2: Move Expectation Inside the Summation (Linearity Assumption)**

$$E\left[\sum_\ell \sum_n s_{\ell n} g_n \varepsilon_\ell\right] = \sum_\ell \sum_n g_n E[s_{\ell n} \varepsilon_\ell]$$

**Step 3: Apply LIE + Share Exogeneity Assumption**

$$E[s_{\ell n} \varepsilon_\ell] = E[s_{\ell n}] E[\varepsilon_\ell | s_{\ell n}] = 0$$

**Step 4: Final Result:** $\sum_\ell \sum_n g_n E[s_{\ell n}] \cdot 0 = 0$

# Identification II: Shifts are Exogenous

- Borusyak, Hull and Jaravel (2022), ReStud
- Identification comes from shifts being uncorrelated w/ $\epsilon_l$
- Example: suppose we want an instrument for wages $x_l$
  - lottery randomly assigns subsidy $g_n$ (shift) to industry $n$.
  - employment growth, $x_l$, instrumented by wt. avg. of subsidies, using initial employment share as weight
  - exclusion restriction: subsidy affects wages by shifting labour demand, not supply
- BHJ (2022): share-weighted average of random shifts is itself as-good-as-random, even shares are endogenous
- In practice: $g_n \perp$ average $\epsilon_l$ across units with weights $s_{ln}$
  - subsidies, even if not truly random, should not vary systematically with $\epsilon_l$

# A practical example from my research

$$X_{id} = \phi \cdot \Delta s_d + \Gamma \cdot \Delta inc_d + \gamma_s + \varepsilon_{ijd}$$

| | (1) Males | (2) Educated | (3) Farm Size | (4) Ag. HH | (5) Landowner |
|---|---|---|---|---|---|
| Δ Wt. Income | 0.078 | 0.235*** | -0.943 | 0.044 | -0.070 |
| | (0.068) | (0.079) | (0.575) | (0.037) | (0.047) |
| Δ Origin Income | Yes | Yes | Yes | Yes | Yes |
| State FEs | ✓ | ✓ | ✓ | ✓ | ✓ |
| N | 38589 | 38589 | 12355 | 38589 | 38588 |
| $R^2$ | 0.021 | 0.020 | 0.103 | 0.051 | 0.088 |

# Extension: what about spillovers?

- If employment in industry $n$ declines after shift, agg. employment may not change
- Solution: specify SSIV at level of region/labour market
  - Captures spillovers when workers move across industries in response to subsidies
- BHJ (2022): use SSIV as a "translation device"!
  - i.e. Reframe SSIV as an IV problem at aggregate level
- Numerical equivalence:

# Numerical Equivalence

**IV Estimator at the Unit Level:** $\hat{\beta} = \frac{\sum_\ell z_\ell y_\ell^\perp}{\sum_\ell z_\ell x_\ell^\perp} = \frac{\sum_\ell \sum_n s_{\ell n} g_n y_\ell^\perp}{\sum_\ell \sum_n s_{\ell n} g_n x_\ell^\perp}$

**Rewriting Using Summation Over Shocks:**

$$\hat{\beta} = \frac{\sum_n g_n \sum_\ell s_{\ell n} y_\ell^\perp}{\sum_n g_n \sum_\ell s_{\ell n} x_\ell^\perp}$$

**Defining Shock-Level Aggregates:** $y_n^\perp = \frac{\sum_\ell s_{\ell n} y_\ell^\perp}{\sum_\ell s_{\ell n}}, \quad x_n^\perp = \frac{\sum_\ell s_{\ell n} x_\ell^\perp}{\sum_\ell s_{\ell n}}$

**Final Shock-Level IV Estimator:**

$$\hat{\beta} = \frac{\sum_n s_n g_n y_n^\perp}{\sum_n s_n g_n x_n^\perp}$$

where $s_n = \sum_\ell s_{\ell n}$ represents the "importance" of shock $n$.

# How do I implement SSIV?

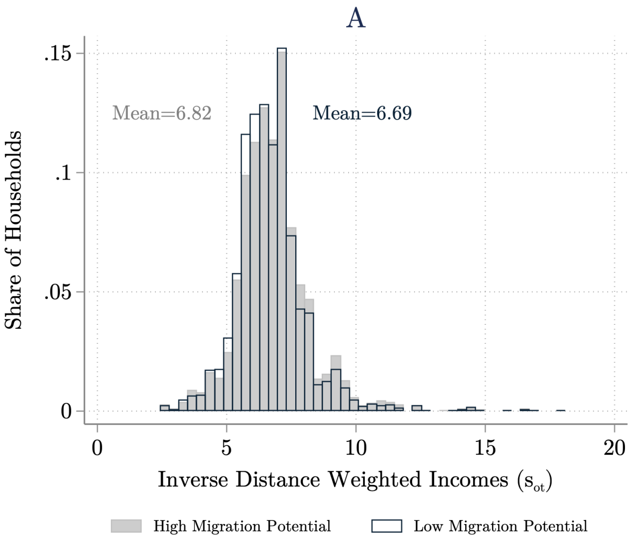
# Checklist for Share-based approach (BHJ, 2024)

1. Explain why shares are suitable IVs
   - should capture exposure to THAT shock only
2. Think about unit-level controls
   - e.g. total migration share, to leverage variation in migrant composition, not high/low intensity
3. Which shares matter most?
   - see Rotemberg weights in GPSS (2020) + bartik_weight command
   - focus on these shares for balance tests
4. Balance tests for individual shares

# Checklist for shift-based approach (BHJ, 2024)

1. How does SSIV approximate idealized experiment?
2. Include incomplete share control, $S_l = \sum_n s_{ln}$
   - Ideally, SSIV is a weighted *average* of random shifts
   - Breaks down when SSIV is a weighted *sum*
3. Lag shares to base or pre-period
4. Balance tests for shifts

# Another Example from my research



# Resources

- A Practical Guide to Shift-Share Instruments, Borusyak et al. (2024) (see FAQ)
- General Equilibrium Effects in Space: Theory and Measurement, Adao et al. (2023)
- Quasi-Experimental Shift-Share Research Designs, Borusyal et al. (2022)
- Bartik Instruments: What, When, Why, and How, Goldsmith-Pinkham et al. (2020)